SEISENSE Journal of Management
Vol 6 No 1 (2023): DOI: https://doi.org/10.33215/vm172083 , 98-122
Research Article

Tobin-Q, Liquidity and Momentum risk-premia: A Demonstration of Weighted Least Squares Regression Approach

Mohammad Azam [1]

[1] Faculty of Management Sciences, National University of Modern Languages, Peshawar Campus, Pakistan

* Corresponding author: azam.ims@hotmail.com

Article History
Received 2023-03-12
Revised 2023-07-25
Accepted 2023-08-02
Published 2023-08-08

Keywords
Tobin-q risk premium
Liquidity risk premium
Momentum risk premium
Fama-French five-factor model
Weighted Least Square (WLS) Regression

Abstract

Purpose - The basic purpose of the study is to examine whether Tobin-q, liquidity and momentum risk-premium contributes the explanatory power in terms of explaining portfolio returns in PSX.

Design/Methodology - The Weighted Least Square (WLS) regression technique is empirically used to examine the nexus between risk-factor and portfolio returns using PSX dataset. The models provide useful tools for making efficient strategies in the jurisdiction of investments and portfolio constructions.

Findings - The study reveals that multidimensional liquidity exhibits weak significant results while Tobin-q and momentum risk-factors demonstrate statistically significant determinants for PSX. Furthermore, WLS regression produces robust coefficient results than OLS regression as except liquidity all the factors exhibit substantially improved results.

Practical Implications - The study findings would be useful for stocks and portfolio managers constructing optimal and diversified portfolios while investing in PSX.

Introduction

The asset pricing models (APMs) have become increasingly prominent since the CAPM of (Sharpe, 1964, Lintner, 1965; Mossin, 1966) has revolutionized the magical specification of asset evaluation technique producing substantially improved results. It is assumed single-factor model and widely used in research with significant performance (Gaytán Cortés, 2023). It is augmented by various anomalies such as, size, value, momentum, profitability, liquidity and investment anomaly. Since Basu (1977) pioneering work on the Price-to-earnings (P|E) ratio as an anomaly played a vital part in the introduction of a new discipline in investment and portfolio management.

The capital asset pricing model (CAPM) is a financial model for calculating an asset's expected return based on associated risk (Vergara-Fernández, Heilmann, & Szymanowska, 2023). The market risk premium-the additional return investors need to invest in a risky asset instead of a risk-free asset-and the expected rate of return on a risk-free asset are both taken into consideration by the model. Based on the notion that investors must be rewarded in two ways-time value of money and risk-is the CAPM. The time value of money is represented by the risk-free rate, and the compensation for risk is the risk premium. The CAPM formula is used to calculate the expected return of an asset, given its level of risk. The formula is:

$$R_i\text{-}R_f = R_f + \beta\ (R_m - R_f) + \varepsilon \qquad (1)$$

Where, Rf is Risk-free rate, Beta is the asset's sensitivity to market risk and the Market Return is the expected return of the overall market.

The most commonly used asset pricing model (henceforth APM) for estimating expected stock/portfolio returns is Fama and French (1993) three-factor model which is an extension of the Capital Asset Pricing Model (CAPM) recommended by Fama and French (1993). The model incorporates three additional factors-market size, book-to-market equity, and firm size-in addition to the traditional market risk factor to explain the cross-section of stock returns. The model suggests that these three factors, in addition to market risk, are priced by the market, and can explain the returns of stocks better than CAPM alone (Oyedeko, Mamidu, & Kolawole, 2023). This model is widely used by practitioners and academics in modern asset pricing and portfolio construction. It is assumed as a benchmark model for explaining cross-sectional stock returns.

Carhart (1997) four factor model (C-4FM) is an asset pricing model that expands on the traditional single-factor CAPM by adding three additional factors to capture market risk. The four factors are the market risk premium, size risk premium, value risk premium, and momentum risk premium. The market risk premium is the traditional risk premium of the CAPM, which measures the risk of investing in the market as a whole. The other three factors measure the risk of investing in certain sectors or types of stocks. The size risk premium measures the risk of investing in small-cap stocks, the value risk premium measures the risk of investing in value stocks, and the momentum risk premium measures the risk of investing in stocks with high momentum. The Carhart (1997) four-factor model is used to estimate the expected return of a security based on its exposure to each of the four factors.

Fama and French (2015) five-factor model is an extension of the three-factor model developed by Fama and French (1993). It adds two additional factors – profitability and investment – to the original three factors of market risk, size risk and value risk. The five-factor model is used to explain the differences in expected returns of various stocks. According to the model, stock returns are related to the five factors as the returns of the stock is related to the returns of the overall market known as market risk; the return of the stock is related to the size of the company. Smaller companies tend to have higher returns than larger companies known as size risk. However, the return of the stock is related to its value. Stocks with a lower price-to-book ratio tend to

have higher returns than stocks with a higher price-to-book ratio known as value risk; the returns of the stock are related to the profitability of the company. Companies with higher profitability tend to have higher returns than companies with lower profitability known as profitability risk and the return of the stock is related to the amount of investments the company makes known as investment risk.

Since then, APMs have undergone significant changes and now play an important role in the decision-making of investors and portfolio managers. Furthermore, Azam and Naveed (2021) produced statistically significant results using data from the Pakistan Stock Exchange (PSX) by augmenting multidimensional liquidity and momentum with Fama and French (2015) five-factor model (seven-factor model). Azam (2022) recently used Tobin-Q to augment various asset pricing models, and the results revealed statistically significant estimates in PSX, but the combination of multidimensional liquidity, momentum and Tobin-q risk factor with Fama and French (2015) five-factor model is not yet investigated particularly in emerging equity market therefore the model's explanatory power in the frontier equity market requires further investigation by integrating multidimensional liquidity, momentum, and Tobin-Q simultaneously with the Fama and French (2015) five-factor model. The study contributes to innovation by utilising these three anomalies in the existing body of knowledge for the first time, distinguishing this study from previous empirical studies on PSX. Furthermore, this study will reveal whether these factors substantially improve the explanatory power of the model and significant coefficients using the Weighted Least Squares regression technique with a view to evaluate empirical robustness in PSX through estimations of various asset pricing models.

More recently, Azam (2023) used 286 non-financial firms' data from PSX employing various augmented APMs between 2006-2022. The findings observed statistically significant results for multidimensional Liu (2006) liquidity as independent as well as mediating variable in the market. Furthermore, based on Gibbons, Ross & Shanken (1969), the liquidity augmented FF5FM is revealed as valid model for explaining portfolio returns in the market.

This study objects to make a two-fold contribution. The weighted least squares (WLS) regression approach is used to investigate the relationship between liquidity, momentum, and Tobin-q risk factors, as well as investment, profitability, value, size, and market-beta factors, and portfolio returns using five to eight-factor asset pricing models. Our empirical method of examining the pricing of multidimensional liquidity, momentum and Tobin-q risk factors leads to our second significant contribution. We specifically focus on Tobin-q, liquidity and momentum adjusted asset pricing models (eight-factor model) to examine whether these risk factors are useful for investors and portfolio managers during decision-making process while investing in PSX.

Table 1: Operational Definition of Risk-Factors

Risk-factor	Description
Market (Rm-Rf)	The market risk premium is measured by the difference between the expected return on a stock and the risk-free return. It is calculated by subtracting the risk-free rate, such as the 1-year Govt. Treasury bills, from the expected return on the stock (Sharpe, 1964, Lintner, 1965; Mossin, 1966).
Size (SMB)	The size risk premium is determined by the market's perception of the risk associated with investing in equities. Moreover, the size risk premium is measured by calculating the difference between the expected return of a portfolio of small-cap stocks and the expected return of a portfolio of large-cap stocks. This risk premium is typically expressed as a percentage and denoted as SMB. The higher the risk premium, the greater the expected return of the small-cap portfolio relative to the large-cap portfolio (Banz 1981).
Value (HML)	However, the value risk premium is measured by calculating the difference between the expected returns of value stocks (higher B|M ratio) and the expected return of growth

	stocks (lower B|M ratio). The risk premium is the reward investors receive for taking on additional risk in the markets. It is often expressed as a percentage and denoted as HML. Moreover, the value premium and the growth premium refer to two different investment strategies that attempt to beat the market. The value premium is an investment strategy that seeks to identify undervalued stocks, while the growth premium is an investment strategy that seeks to identify stocks with the potential for higher-than-average growth (Chan, Hamao & Lakonishok 1991; Fama & French, 1993).
Profitability (RMW)	The profitability risk premium is measured by calculating the difference between the expected return of stock having greater operating profitability and the expected return of stock having lower operating profitability (Fama & French, 2015).
Investment (CMA)	The investment risk premium is measured by calculating the difference between the expected return of a stock having higher growth in assets and the expected return of a stock having lower growth in assets. This difference is known as the investment risk premium and can be expressed as a percentage and denoted as CMA (Fama & French, 2015).
Momentum (WML)	The momentum factor measures the rate of change in the price of a security over time. It is calculated by subtracting the current price of a security from the price of the same security over a specified time period. The outcome is then divided by the original price. The momentum factor is used to identify a security's market trend and to assist investors in identifying potential entry and exit points (Jegadeesh, 1990; Jegadeesh & Titman, 1993; Carhart, 1997).
Liquidity (IML)	The multidimensional liquidity is a concept used to evaluate the liquidity of an asset or security. It is also known as multi-factor liquidity. It takes into account multiple factors, such as the size of the order, the speed at which it can be executed, the cost of executing the order, the availability of buyers and sellers, and the cost of holding the asset or security. This concept is used to assess the liquidity of both traditional and alternative assets. A higher degree of multidimensional liquidity means that an asset or security can be bought or sold more quickly and at a better price, making it easier and more efficient to trade (Liu (2006).
Tobin-q (UMO)	Tobin q measures Tobin's q, also known as the q-ratio, is a measure of the market value of a company relative to the replacement cost of its assets. It is calculated by dividing the market value of a company's assets, usually the total market value of outstanding shares, by the replacement cost of the company's assets. It is named after the Nobel Prize-winning economist James Tobin, who introduced it in 1969. The q-ratio is used to measure the efficiency of a firm's investment decisions, and to measure the relative value of a company's stock. The q-ratio can also be used to compare the relative value of different companies in the same industry (Tobin-q, 1969; Azam, 2022c).

Literature Review

A plethora of prior literature of APMs consist of various augmented risk-factors such as momentum, liquidity, leverage is invested and observed better estimates the expected return of stocks while using time-series ordinary least square regression technique (Fama and French, 1993; 2015) and cross-sectional two-steps regressions technique (Fama & MacBeth, 1973) around the globe. The literature addresses how momentum and contrarian techniques can affect future anomalous returns. In the developed stock markets, research has been done on momentum (1–12 month) and contrarian (3-5 year) strategies. However, diversified evidences for the momentum factor were observed in developed equity markets. Atilgan et al. (2022) used momentum strategies and observed statistically significant and positive momentum effect using merging equity markets. The findings

also reveal that the momentum strategy consistently outperforms local market indexes. Hurn and Pavlov (2003) conducted a study on the effects of momentum only for big stocks. They found results for the factor in the Australian market to be statistically significant. Significant outcomes with regard to the momentum factor were also documented by Stork, (2008); Demir et al., (2004). On the other hand, Durand et al. (2011) did not find any reliable findings for the momentum factor utilising daily data over the years 1980–2001. However, the valid proof of the significance of the momentum factor in comparison to other determinants was provided by (Demir et al, 2004). As a result, there are contrasting viewpoints for momentum premium; therefore, the momentum and portfolio stock returns nexus needs to be tested in emerging market of Pakistan.

Moreover, Chiah et al. (2016) examined the 5FM using Australian equity market data and observed outperformance as compare to 3FM. In addition, Lohano and Kashif (2018) examined the 5FM using 896 enlisted firms' data from PSX. Using time span from Nov-2000 to Dec-2016, the cross-sectional analysis has been conducted and observed significant findings of 5FM. However, Chiah et al. (2016) examined the 5FM using Australian equity market data and found that it outperformed the 3FM. Furthermore, Lohano and Kashif (2018) investigated the 5FM using PSX data from 896 enlisted firms. The cross-sectional analysis was conducted from November 2000 to December 2016 and found significant findings of 5FM. On the other hand, Khan et al. (2021) examined momentum strategy using 466 enlisted companies' dataset from PSX. The results reveal statistically significant but inverse nexus momentum and portfolio returns for the time span between 2009 and 2017. Ali et al. (2020) examine the demutualization and liquidity nexus in PSX using Turnover, Amivest ratio and Bid-Ask Spread proxies of liquidity. The data of 137 non-financial firms are used using panel data analysis for time span from 2005 to 2017. The findings reveal that demutualization substantially impacts the liquidity and indirectly mitigate the transaction cost in PSX.

In addition, Azam (2022c) pioneered and evaluated Tobin-q as a risk premium augmented with CAPM, 3FM, C-4FM, and 5FM more recently using the PSX dataset across a 27-year span, from 1994 to 2020. Using monthly data from 521 financial and non-financial firms, they conducted a thorough analysis on PSX using the time-series OLS regression approach. Even when the market and investments show insignificant returns, the statistics show statistically significant parameters like size, value, profitability, and Tobin-q risk factor. According to the GRS test, Tobin-q enhanced 5FM was the most productive model on the market. More recently, Ahmad et al. (2023) used the Tobin-q as an indicator of firm financial performance and evaluated it as a dependent variable, whereas this study uses Tobin-q as an independent variable that is categorized as firms with undervalued Tobin-q outperform firms with overvalued Tobin-q, and is thus used as an independent variable (undervalued minus overvalued). Dirkx and Peter (2020) used momentum as additional factor augmented with 5FM and observed highly significant findings for German stock market.

Shi (2023) examined the performance of liquidity augmented FF-5FM (L-5FM) using China's A-share Market. Using grouping and regression analysis the findings reveal significant liquidity premium in the market. Furthermore, the L-5FM outperforms the 5FM in terms of explanatory power of the model. Kalim, Saeed, & Kamil (2023) used manufacturing industry data and observed size has no significant impact on companies' profitability. However, the sales growth has significant nexus with firms' profitability in PSX. Azam (2022c) observed highly statistically significant findings for Tobin-q while Azam and Naveed (2021) discovered statistically significant findings for multidimensional liquidity and momentum factors for PSX augmented with 5FM. Whereas past research indicates that 5FM is augmented by Tobin-q, liquidity, and momentum individually, the combination of these three elements with 5FM is still scarce, particularly in emerging equities markets such as PSX. Therefore, this study strives to fill the gap and for further robustness uses WLS regression. Consequently, based on the above literature discussed above, the study withdraws the following hypotheses to be tested using PSX dataset:

H1: Tobin-q/Liquidity/Momentum/Market-Beta/Size/Value/Profitability/Investment-risk premium (TLM-MSVPI) has significant nexus with portfolio returns.

Model Specification

Based on WLS regression, this study employs the following nested and augmented asset pricing models:

1. Fama & French (2015) five-factor model (5FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_p(RmW) + \beta_i(CmA) + \varepsilon_i \quad (4)$$

Where, $R_i - R_f$, is excess returns of portfolio, $R_m - R_f$, is the excess returns of market, SmB is the Small Minus Big firms returns called Size factor, HmL is the High minus Low firms returns called Value factor, RMW is the Robust Minus Weak firms returns called Profitability factor, CMA is the Conservative Minus Aggressive firms returns called Investment factor and β_m, β_s, β_v, β_p, and β_i are the coefficients of market, size, value, profitability and investment factors respectively.

2. Azam (2021) Liquidity augmented six-factor model (A-6FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_p(RmW) + \beta_i(CmA) + \beta_l(ImL) + \varepsilon_i \quad (5)$$

Where, IML is the Illiquidity Minus Liquidity firms' returns called multidimensional liquidity factor, and β_m, β_s, β_v, β_p, β_i and β_l are the coefficients of market, size, value, profitability, investment and liquidity factors respectively.

3. Azam (2022c) Tobin-q augmented six-factor model (A-6FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_p(RmW) + \beta_i(CmA) + \beta_t(UmO) + \varepsilon_i \quad (6)$$

Where, IML is the Illiquidity Minus Liquidity firms' returns called multidimensional liquidity factor and β_m, β_s, β_v, β_p, β_i and β_t are the coefficients of market, size, value, profitability, investment and Tobin-q factors respectively.

4. Fama & French (2018) six-factor model (6FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_p(RmW) + \beta_i(CmA) + \beta_w(WmL) + \varepsilon_i \quad (7)$$

Where, IML is the Illiquidity Minus Liquidity firms' returns called multidimensional liquidity factor and β_m, β_s, β_v, β_p, β_i and β_w are the coefficients of market, size, value, profitability, investment and momentum factors respectively.

5. Azam (2021) Seven-factor model (Liquidity and Momentum augmented Fama & French (2015) model (A-7FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_p(RmW) + \beta_i(CmA) + \beta_l(ImL) + \beta_w(WmL) + \varepsilon_i \quad (8)$$

Where, IML is the Illiquidity Minus Liquidity firms' returns called multidimensional liquidity factor and β_m, β_s, β_v, β_p, β_i, β_l and β_w are the coefficients of market, size, value, profitability, investment, liquidity and momentum factors respectively.

6. Liquidity and Tobin-q augmented Fama & French (2015) seven-factor model (LT7FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_p(RmW) + \beta_i(CmA) + +\beta_l(ImL) + \beta_t(UmO) + \varepsilon_i \quad (9)$$

Where, IML is the Illiquidity Minus Liquidity firms' returns called multidimensional liquidity factor and β_m, β_s, β_v, β_p, β_i, β_l and β_t are the coefficients of market, size, value, profitability, investment, liquidity and Tobin-q factors respectively.

7. Momentum and Tobin-q augmented Fama & French (2015) seven-factor model (LT7FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_p(RmW) + \beta_i(CmA) + \beta_w(WmL) + \beta_t(UmO) + \varepsilon_i \quad (10)$$

Where, IML is the Illiquidity Minus Liquidity firms' returns called multidimensional liquidity factor and β_m, β_s, β_v, β_p, β_i , β_w and β_t are the coefficients of market, size, value, profitability, investment, momentum and liquidity factors respectively.

8. Liquidity, Momentum and Tobin-q augmented Fama & French (2015) eight-factor model (LMT8FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SmB) + \beta_v(HmL) + \beta_p(RmW) + \beta_i(CmA) + \beta_l(ImL) + \beta_w(WmL) + \beta_t(UmO) + \varepsilon_i \quad (11)$$

Where, IML is the Illiquidity Minus Liquidity firms' returns called multidimensional liquidity factor and β_m, β_s, β_v, β_p, β_i, β_l, β_w and β_t are the coefficients of market, size, value, profitability, investment liquidity, momentum and Tobin-q factors respectively.

9. The comparison of Model using Explanatory Power

Following Hua (2022), this study uses the 5FM as the benchmark to compare with. However, other models are augmented by Tobin-q, liquidity and momentum factors separately and together to test the performance of the models mentioned in Equation (5 to 11).

10. Model Performance test using GRS (Wald version) Test

The Wald test is a statistical test used to evaluate a null hypothesis and an alternate hypothesis. The test is based on the Wald statistic, which measures the discrepancy between the expected value under the null hypothesis and the observed value of a test statistic. The Wald test is used to test the significance of model parameters, such as the slope and intercept in a linear regression model. It is also used to check for differences between two population means, or to test the difference between observed and expected frequencies in a contingency table. However, the Gibbons, Ross, and Shanken (GRS) test determines whether the returns on a portfolio of assets are statistically significant. The test assumes that the portfolio's returns can be divided into two parts: the expected return of the portfolio based on the expected returns of the individual assets, and the unexpected return of the portfolio, which is the difference between the actual return and the expected return. The GRS test compares the unexpected return to the expected return to determine its significance. The GRS test indicates that the portfolio's returns are not simply due to chance if the difference is statistically significant. Following is the GRS specification:

$$GRS = \left(\frac{T}{N}\right)\left(\frac{T}{T}\ \frac{-N-L}{-L-1}\right)\left[\frac{\hat{\alpha}'\,\hat{\Sigma}^{-1}\,\hat{\alpha}}{1+\bar{\mu}!\,\hat{\Omega}^{-1}\hat{\mu}}\right] \sim F(N,\ T-N-L) \quad (12)$$

where, $\hat{\alpha}$ = N x 1 estimated constant term vector, $\hat{\sum}$ = Stochastic terms unbiased covariance matrix, $\bar{\mu}$ = L x 1 factor portfolio average matrix, $\hat{\Omega}$ = Factor portfolio unbiased covariance matrix, T = No. of observations, N = No. of regression equations and L = No. of factors in the regression. Using the above specification, this study investigates the GRS-Wald version F-test based on the following hypothesis:

$$H_0: \alpha_i = 0 \text{ i: } 1, 2, 3, \ldots, N.$$

where, the GRS-F test denotes that all alpha coefficients are equal to zero (α=0)

$$H_1: \alpha_i \neq 0 \text{ i: } 1, 2, 3, \ldots, N.$$

where, the GRS-F test denotes that all alpha coefficients are not equal to zero ($\alpha \neq 0$)

Data and Methodology

This research spans 354 months, beginning in July 1993 and ending in December 2022. Following Barber and Lyon (1997), this study examines the market and the performance of asset pricing models using 522 financial and non-financial firms' data. Using five to eight-factor asset pricing models, the weighted least squares (WLS) regression technique is utilised to study the link between liquidity, momentum, Tobin-q risk factors, as well as investment, profitability, value and size anomalies and portfolio stock returns.

1. Weighted Least Square Regression

In terms of robustness, the Weighted Least Square (WLS) regression is better than Ordinary Least Square (OLS) regression in cases when the data points have different variances due to the different sampling or experimental conditions. In such cases, the errors are not normally distributed but have a different variance. WLS regression is more robust to stock returns type of data and gives better and robust results. Furthermore, WLS regression is a regression analysis technique in which different weights are assigned to different data points. It is used in data analysis to account for heteroscedasticity, which is the presence of unequal variance among data points. The technique gives more weight to data points with lower variance and less weight to those with higher variance. This reduces the impact of outliers and the effect of heteroscedasticity in the data which the stock returns usually face. WLS regression can also be used to assign different weights to different types of data points, such as those from different time periods or geographic regions. WLS regression can also be used to assign different weights to different types of data points, such as those from different time periods or from different areas. Therefore, this study employs the WLS to produce more robust estimates and produce more valid results using multiple asset pricing models in PSX.

2. Portfolio Construction Risk-factor Measurement

The study constructs 25 value-weighted portfolios by following Fama and French (1993; 2015) based on Market-cap (Big, 4, 3, 2 and Small) and Book-to-Market (B|M) ratio (High, 4, 3, 2 and Low), as follows:

Table 2: Portfolio Construction Matrix

S/B|M	H_B|M	4	3	2	L_B|M	S/B|M	H_B|M	4	3	2	L_B|M
Big	BH	B4	B3	B2	BL	**Big**	SBM1	SBM2	SBM3	SBM4	SBM5
4	4H	44	43	42	4L	**4**	SBM6	SBM7	SBM8	SBM9	SBM10
3	3H	34	33	32	3L	**3**	SBM11	SBM12	SBM13	SBM14	SBM15
2	2H	24	23	22	2L	**2**	SBM16	SBM17	SBM18	SBM19	SBM20
Small	SH	S4	S3	S2	SL	**Small**	SBM21	SBM22	SBM23	SBM24	SBM25

Notes: Table 2 shows the portfolio construction matrix which combines size and value pattern. The BH denotes the stocks having Big market-cap and High B|M ratio. Similarly, BL denotes the stocks having Big market-cap

and Low B|M ratio. In the same way, SH denotes the stocks having Small market-cap and High B|M ratio and SL denotes the stocks having Small market-cap and Low B|M ratio.

Table 3: Risk-factor Measurement

Risk-factor	Measurement
Market (Rm-Rf)	The market risk premium is measured by the difference between the expected return on a market portfolio and the risk-free rate. It is used to calculate the required return on any individual equity investment.
Size (SMB)	The size risk premium is a measure of the risk associated with investing in small-cap stocks. It is often used by investors to compare the expected return of small-cap stocks relative to the expected return of larger, more established stocks.
Value (HML)	The value premium is based on the belief that stocks that are undervalued in the market will eventually rise to their "true" value, while the growth premium is based on the belief that stocks with the potential for higher-than-average growth will eventually outperform the market. It is measured by value average portfolios returns minus growth average portfolio returns.
Profitability (RMW)	The operating profitability is measured as (revenue – cost of goods sold – interest expense – selling and admin expenses) divided by Book-value.
Investment (CMA)	Investment is measured with the growth in total assets and is measured as total assets of this year divided by total assets of previous year.
Momentum (WML)	Momentum indicators can also be used to compare the performance of various securities. It is measured as average returns of previous 12-months. The positive returns are considered the Winner firm/portfolio and the negative returns are considered the Loser firm/portfolio. To calculate momentum risk-premium, winner minus loser firm/portfolio.
Liquidity (IML)	Liu (2006) multidimensional Liquidity is used to measure the liquidity risk-premium using the equation (2). It focuses on the velocity of trading which was not considered in previous studies. Liu (2006) pioneer the multidimensional liquidity notion using the following specification: $LIQ = \left[\mathrm{X} + \frac{\frac{1}{\mathrm{Z}}}{11.000}\right] \mathrm{x} \frac{21\mathrm{x}12}{\mathrm{Y}}$ (2) Where, X = No. of days without trading in the past 1 year, Y = No. of days with trading in the market, Z = Mean turnover in the past 1 year, extracted from the sum of the daily turnovers in the past 1 year; with the daily turnover being the ratio of the number of outstanding stocks at the end of that day.
Tobin-q (UMO)	Tobin-q can be measured by dividing the market value of a firm by the replacement cost of the firm's assets. The ratio is used to measure the amount of capital allocated to a company relative to the amount of capital that would be required to replace the company's assets at current prices. If the ratio is greater than one, it indicates that the market values the company's assets more than their replacement cost, while a ratio of less than one indicates that the market values the company's assets less than their replacement cost. Tobin's q ratio is used as an indicator of corporate performance, as it reflects how well a company is able to use its assets to generate profits. It can also be used to assess the valuation of a company relative to its peers. Companies with higher Tobin's q ratios are often seen as being more valuable than those with lower ratios. Tobin's Q = [(book value of assets + market value of equity)–(book value of equity)] / (book value of total assets) (3)

Results and Discussion

In this part, we analyse the results extracted from weighted least squares regression using various asset pricing models including multidimensional liquidity, momentum and Tobin-q augmented Fama and French (2015) five-factor model for PSX.

Table 4: Descriptive Statistics and Correlation Matrix

Variable	RmRf	SMB	HML	RMW	CMA	WML	IML	STQ
RmRf	1	-0.0766	0.0133	0.0178	-0.0024	-0.0842	-0.1051	0.0199
SMB	-0.0766	1	0.0429	-0.0337	0.0759	0.2443	0.2849	0.3124
HML	0.0133	0.0429	1	-0.2479	0.2557	-0.2517	-0.2114	-0.0995
RMW	0.0178	-0.0337	-0.2479	1	0.1198	0.0891	0.0815	-0.0268
CMA	-0.0024	0.0759	0.2557	0.1198	1	0.2658	0.1088	0.025
WML	-0.0842	0.2443	-0.2517	0.0891	0.2658	1	0.4664	-0.1353
IML	-0.1051	0.2849	-0.2114	0.0815	0.1088	0.4664	1	-0.2144
STQ	0.0199	0.3124	-0.0995	-0.0268	0.025	-0.1353	-0.2144	1
Variable	**RmRf**	**SMB**	**HML**	**RMW**	**CMA**	**WML**	**IML**	**STQ**
Mean	0.00892	-0.0023	-0.0018	0.00227	0.00139	0.01125	0.0035	-0.0017
Std. Dev.	0.08316	0.03218	0.01865	0.01573	0.01496	0.02783	0.03463	0.01815
Min	-0.4502	-0.1536	-0.0944	-0.0582	-0.0443	-0.1404	-0.1376	-0.0774
Max	0.24458	0.12971	0.12753	0.05481	0.06083	0.19592	0.16508	0.06479
Obs.	360	360	360	360	360	360	360	360

Notes: Table 4 presents the correlation matrix between independent factors and descriptive statistics of factors which includes average, standard deviation, minimum and maximum values of each factor using in this study.

Table 4 displays two parts of the study which consists of descriptive statistics and correlation matrix. The study's two components, a correlation matrix and descriptive statistics, are shown in Table 4 as two separate sections. Market excess returns have negative correlations with size, investment, momentum, and liquidity (-0.0766, -0.0024, -0.0842, and -0.1051, respectively) when looking at the magnitude of these determinants. Value exhibits negative association with profitability, momentum, liquidity, and Tobin-Q, whereas size exhibits negative nexus with profitability (-0.0337). Similarly, Tobin-q has inverse correlations (-0.0268, -0.1353 and -0.2144) with profitability, momentum, and liquidity. Momentum and liquidity have the largest positive correlation (0.6648), which is a modest effect and may not lead to multicollinearity when both factors are used in the same model. The descriptive statistics of all independent factors used in this study for analysis, including Tobin-q, liquidity, momentum, investment, profitability, value, size anomalies, and market risk factor, are shown in the second section of Table 4. The average returns for size, value and Tobin-q are all negative (-0.00229, -0.00177 and -0.00172) with standard deviations of (0.032182, 0.018648 and 0.018153) respectively. The other factors demonstrate positive mean returns for the sample period used in this study. The results show that WLS regression produces more valid and robust results than Azam (2021; 2022), who uses OLS regression on a nearly identical PSX dataset.

Table 5: Fama-French (2015) five-factor model Estimation of Coefficients based on weighted least squares (WLS) regression

RmRf	H_B|M	4	3	2	L_B|M	t-stat	H_B|M	4	3	2	L_B|M
Big	-0.378***	-0.469***	-0.109***	-0.048	-0.006	**Big**	-7.738	-17.770	-4.534	-1.013	-0.226
4	0.629***	-0.384***	-0.011	0.077*	0.874***	**4**	17.955	-9.019	-0.422	1.737	21.578
3	0.492***	0.452***	0.683***	0.357***	0.508***	**3**	6.482	5.903	18.409	8.477	4.060
2	0.191***	0.004	-0.214***	1.202***	0.618***	**2**	8.859	0.067	-3.781	14.994	5.473
Small	0.278***	0.031	-0.077*	0.253***	-0.155***	**Small**	5.456	1.563	-1.944	4.582	-5.660
SMB	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-1.711*	-1.324***	-1.043***	-0.353	-1.496***	**Big**	-1.941	-3.880	-4.263	-0.847	-4.799
4	-0.534*	0.993***	-0.678***	-2.146***	-0.057	**4**	-1.940	2.665	-3.654	-3.654	-0.082
3	2.641***	0.903***	-0.272**	0.237	0.038	**3**	7.662	8.004	-2.385	0.696	0.197
2	-0.919	-0.263	-0.208	-0.454*	0.334	**2**	-1.519	-1.562	-0.558	-1.835	1.461
Small	1.193***	0.451*	0.081	-1.109**	0.453***	**Small**	4.944	1.893	0.407	-2.181	2.966
HML	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	6.528***	1.727***	1.823***	0.901***	0.057	**Big**	8.260	4.488	9.490	3.924	0.388
4	0.542	0.631	0.543***	-0.620	0.566	**4**	1.101	0.934	6.506	-0.657	0.963
3	-1.611	0.865**	-0.292**	0.334	-2.612***	**3**	-0.971	2.036	-2.275	1.183	-4.195
2	0.493	2.300***	-1.864***	-1.322***	-1.991***	**2**	0.458	8.255	-7.360	-2.944	-6.924
Small	1.863***	0.482	-0.283	-1.874***	0.956***	**Small**	4.927	1.028	-1.262	-6.157	5.009
RMW	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	3.313***	-0.195	0.343*	0.562***	-0.355	**Big**	6.445	-0.862	1.948	2.845	-1.523
4	-2.107***	-1.025*	-0.686***	-1.056	-1.472*	**4**	-11.367	-1.708	-4.800	-1.543	-1.864
3	-3.730***	1.119**	0.547	-1.218***	-0.913	**3**	-3.172	2.525	1.214	-6.044	-1.363
2	1.422**	2.697***	0.789**	-5.251***	-1.160***	**2**	2.182	22.894	2.429	-8.310	-2.914
Small	1.109***	-1.484***	1.203***	-2.907***	1.008***	**Small**	5.609	-5.012	3.365	-8.513	2.690
CMA	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-1.152***	0.861***	-0.496***	-0.586**	0.174*	**Big**	-4.267	11.275	-4.709	-2.240	1.770
4	-2.110***	-1.753***	-0.714***	0.613	0.784	**4**	-12.193	-4.758	-4.668	0.803	1.649
3	4.763***	1.725**	-0.224	-0.961**	-1.692**	**3**	8.027	2.191	-0.275	-2.517	-2.472
2	2.949***	0.320	1.386***	0.021	-0.008	**2**	11.266	1.096	10.191	0.052	-0.020
Small	0.961***	0.919***	1.374***	-1.025**	-1.461***	**Small**	3.831	4.777	7.780	-1.995	-8.370
Alpha	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.022*	-0.014**	0.002	0.002	0.000	**Big**	-1.677	-2.106	0.573	0.211	0.016
4	-0.025**	0.016	-0.004	-0.008	-0.033**	**4**	-2.027	1.243	-0.812	-0.370	-2.298
3	0.021	-0.013	-0.007	0.004	-0.058*	**3**	0.645	-0.809	-0.561	0.231	-1.929
2	-0.004	-0.030***	-0.024***	-0.031*	-0.031**	**2**	-0.312	-2.884	-2.994	-1.745	-2.241
Small	-0.045***	-0.024***	-0.021***	-0.042***	-0.014**	**Small**	-3.600	-4.485	-3.066	-3.832	-2.046
R-2.	**H_B|M**	**4**	**3**	**2**	**L_B|M**						
Big	0.819	0.573	0.371	0.473	0.144						
4	0.890	0.589	0.333	0.944	0.740						
3	0.748	0.635	0.750	0.538	0.696						
2	0.888	0.771	0.873	0.625	0.400						
Small	0.164	0.271	0.479	0.664	0.259						

Notes: *** and * denote statistical significance levels of 1% and 10%, respectively. Table 5 displays the estimated coefficients, t-statistics (right side), and R-square for each portfolio obtained from the multivariate WLS regression with 25 value-weighted portfolios using 5FM.

Using WLS, the size-factor estimates statistically significant coefficients for 16/25 portfolios but findings provide weak size-effect as small firms' portfolios are significant for 5/10 while 8/10 for big firms' portfolios for the market. The value-factor also shows better estimates as 15/25 portfolios demonstrates statistically significant findings which validate that value-factor is not redundant for PSX while H-B|M and L-B|M ratio portfolios show 5/10 and 6/10 significant portfolios respectively. Moreover, the profitability-factor reveal 20/25 portfolios statistically significant based on t-statistics and for small-firms' portfolios it shows 10/10

significant though H-B|M portfolios also show 9/10 portfolios significant results as well. Similarly, investment-factor also reveal 19/25 portfolios statistically significant results while it also proves 9/10 portfolios significant results for H-B|M ratio portfolios. Furthermore, the R-square ranges from 15% to 94% for portfolio BL and 42 respectively. The results demonstrate that, when compared to Azam (2021; 2022), who uses OLS regression with a nearly identical dataset from PSX, WLS regression yields more reliable and robust results.

Table 6: Liquidity augmented Fama-French (2015) five-factor model (L5FM) Estimation of Coefficients based on weighted least squares (WLS) regression

RmRf	H_B|M	4	3	2	L_B|M	t-stat	H_B|M	4	3	2	L_B|M
Big	-0.074***	-0.031**	0.206***	0.098***	1.017***	**Big**	-2.741	-2.090	6.427	4.054	20.424
4	-0.140***	-0.194***	-0.131***	0.091***	0.062	**4**	-5.354	-11.507	-6.579	4.890	1.444
3	-0.189***	-0.615***	-0.202***	-0.388***	-0.313***	**3**	-7.268	-12.400	-23.055	-15.493	-11.164
2	0.155***	-0.520***	-0.028	-0.007	-0.029*	**2**	6.702	-19.887	-0.456	-0.188	-1.679
Small	0.002	0.765***	-0.012	-0.025	0.334***	**Small**	0.056	19.173	-0.256	-1.284	10.788
SMB	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-1.290***	-0.339	-0.393	0.794***	2.123***	**Big**	-3.067	-1.357	-1.379	3.403	3.498
4	-1.642***	-1.129***	-0.017	-1.693***	0.171	**4**	-12.587	-7.836	-0.127	-8.491	0.603
3	1.145***	-0.123	-0.095**	-0.496***	0.184	**3**	8.924	-0.485	-2.383	-4.886	1.318
2	1.112***	0.120	0.347	0.716**	0.840***	**2**	3.060	0.399	0.751	2.185	3.004
Small	0.343	-2.062**	-0.372	1.269***	0.912***	**Small**	1.198	-2.237	-1.109	4.152	3.000
HML	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.794***	-0.014	-0.803***	-1.701***	1.732***	**Big**	-2.609	-0.061	-6.326	-4.690	3.099
4	-1.453***	2.495***	0.064	0.110	-0.196	**4**	-5.682	8.715	0.399	0.121	-0.360
3	1.426***	2.552***	0.817***	1.125***	-2.083***	**3**	8.417	8.220	3.789	2.618	-6.926
2	1.006***	1.107***	-0.251*	1.789***	1.613***	**2**	3.348	6.399	-1.718	3.536	4.198
Small	1.524***	5.617***	0.148	-1.082***	-0.584	**Small**	7.599	5.990	0.458	-4.519	-1.538
RMW	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-1.829***	-2.258***	0.542**	0.427*	-0.952**	**Big**	-5.740	-8.214	2.514	1.708	-2.025
4	-0.851***	-1.262***	-0.314**	0.835**	-0.323	**4**	-4.013	-3.325	-2.369	2.181	-0.619
3	-0.398	1.770***	-0.978***	0.702***	-0.089	**3**	-1.519	3.730	-3.608	3.598	-0.403
2	0.588**	0.859***	2.353***	-0.071	-0.931***	**2**	1.969	4.411	8.212	-0.157	-5.902
Small	0.413***	1.625**	0.338	-1.702***	1.356**	**Small**	2.778	2.063	0.767	-12.162	2.507
CMA	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	2.251***	1.844***	1.835***	1.570***	-2.904***	**Big**	12.360	13.862	9.988	4.218	-5.980
4	1.542***	-0.009	0.676**	1.500**	1.433***	**4**	6.418	-0.023	2.310	2.023	3.023
3	1.756***	1.933*	1.056*	2.690***	0.328**	**3**	6.428	1.656	1.826	6.116	2.500
2	0.859***	0.260	-1.019***	-2.153***	-2.444***	**2**	4.596	0.561	-9.082	-4.938	-14.372
Small	-0.304	1.006	-1.562***	1.121***	1.982***	**Small**	-0.766	1.443	-4.733	4.085	6.959
IML	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-1.549**	-0.742*	-0.953**	-1.092***	0.521	**Big**	-2.464	-1.704	-2.173	-2.960	0.769
4	-0.483	-0.558	-1.651***	-0.622	-2.284***	**4**	-0.893	-1.341	-6.509	-0.877	-4.473
3	-1.224***	-1.011	-1.056***	-0.361	-1.404***	**3**	-2.833	-1.378	-3.957	-0.754	-3.997
2	-1.555***	-0.526	-0.888**	-1.547***	-2.352***	**2**	-4.552	-1.297	-2.268	-2.782	-4.916
Small	-1.739***	2.474***	-1.182***	-1.199***	-0.143	**Small**	-4.827	2.836	-3.096	-3.729	-0.506
Constant	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.007	-0.023***	-0.032***	-0.017**	0.000	**Big**	0.984	-3.735	-5.328	-2.552	0.041
4	-0.017	-0.014*	-0.011**	-0.007	-0.007	**4**	-1.534	-1.883	-2.070	-0.656	-0.920
3	-0.013**	0.004	-0.013*	-0.003	-0.003	**3**	-2.362	0.281	-1.798	-0.282	-0.847
2	0.002	-0.000	-0.023***	0.012	-0.004	**2**	0.524	-0.005	-3.850	1.268	-0.435
Small	-0.013	-0.009	-0.005	-0.023***	-0.034***	**Small**	-1.548	-0.776	-0.636	-5.990	-3.646
R-squared	**H_B|M**	**4**	**3**	**2**	**L_B|M**						
Big	0.652	0.851	0.787	0.540	0.765						
4	0.765	0.929	0.654	0.907	0.935						

3	0.775	0.828	0.839	0.894	0.752
2	0.696	0.982	0.818	0.543	0.829
Small	0.498	0.822	0.833	0.610	0.443

Notes: *** and * denote statistical significance levels of 1% and 10%, respectively. Table 6 displays the estimated coefficients, t-statistics (right side), and R-square for each portfolio obtained from the multivariate WLS regression with 25 value-weighted portfolios using L5FM.

Table 6 depicts the multidimensional liquidity augmented 5FM (L5FM) using weighted least squares regression approach. In this table, the impact of multidimensional liquidity plus MSVPI on portfolio returns has been analyzed. As the results evidenced that the CAPM (market-risk premium) demonstrates highly statistically significant nexus with portfolio excess returns except six portfolios (SH, 23, S3, 22, S2 and 4L with coefficient = 0.002, -0.028, -0.012, -0.007, -0.025and 0.062 respectively) which stands insignificant based on t-statistics. The size-factor exhibits 15 out of 25 portfolio coefficients statistically significant but negative findings based on t-statistics criteria though small firms' portfolios 6 out of 10 show significant findings, displaying moderately significant effect of size-factor in the market. However, the value factor has 19 out of 25 portfolio coefficients that are significant, which supports the theory by demonstrating that value outperforms growth portfolios in the market. The results also show a negative effect for 9 coefficients. The profitability-factor, on the other hand, has statistically significant coefficients for 20 of the 25 portfolios. Similarly, the investment factor produces statistically significant results in 21 of 25 portfolios. Finally, liquidity-factor shows 17 out of 25 coefficients statistically significant but negative relationship with portfolio returns. In conclusion, only the size-risk premium is moderately significant, while all other factors show significant results using the weighted least square technique for the PSX. Furthermore, the R-square ranges from 44% to 98%. The results show that WLS regression produces more valid and robust results than Azam (2021; 2022), who uses OLS regression on a nearly identical PSX dataset. The adjusted R-square ranges from 44% to 98% indicating substantially improved explanatory power for the L5FM.

Table 7: Tobin-q augmented Fama-French (2015) five-factor model (T5FM) Estimation of Coefficients based on weighted least squares (WLS) regression

RmRf	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.344***	-0.262***	-0.044*	0.053**	0.071***	**Big**	12.928	-14.837	-1.653	2.347	2.780
4	0.065**	-0.098***	-0.058*	-0.064	-0.116***	**4**	2.004	-3.113	-1.730	-0.996	-4.677
3	-0.293***	0.323***	-0.031	0.080***	0.034	**3**	-8.352	4.400	-0.769	3.714	0.707
2	-0.090***	-0.108**	-0.267***	0.150**	0.092**	**2**	-3.063	-2.385	-5.321	2.418	2.270
Small	0.053	0.096**	-0.087**	0.060	-0.291***	**Small**	1.351	2.100	-2.051	0.718	-8.447
SMB	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.150	-1.784***	-1.136***	-0.262	-1.334***	**Big**	-0.269	-7.409	-4.476	-0.815	-4.406
4	-0.916***	-1.272***	-0.751***	-2.522***	-0.506*	**4**	-6.110	-5.909	-3.577	-4.029	-1.830
3	0.284**	-0.567***	-1.274***	0.281	0.410***	**3**	2.423	-6.805	-11.347	1.233	5.182
2	-1.009	-0.036	-0.692**	1.055***	1.452***	**2**	-1.503	-0.224	-2.082	4.119	10.902
Small	-0.316	-0.243	-0.208	-0.741	0.142	**Small**	-1.443	-0.491	-1.124	-0.660	0.931
HML	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	1.638***	0.135	0.683***	-0.514**	0.504***	**Big**	3.909	0.547	4.028	-1.989	3.220
4	-0.563***	0.720**	1.406***	-2.086**	0.417*	**4**	-2.657	2.088	7.512	-2.060	1.739
3	0.290	0.323	2.068***	-0.196	-0.196	**3**	0.675	0.864	11.444	-0.911	-0.763
2	-0.606	0.570**	-0.986***	5.406***	-0.173	**2**	-0.539	2.155	-3.670	11.012	-0.623
Small	1.415***	3.732***	-0.520**	-4.413***	-0.655***	**Small**	4.088	4.218	-2.497	-5.442	-3.427
RMW	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.830**	-0.374*	-1.238***	-0.493***	0.096	**Big**	-2.081	-1.739	-6.984	-3.748	0.389
4	-0.753***	-0.467	0.168	-1.921**	-1.132***	**4**	-5.887	-1.216	0.758	-2.422	-3.172
3	-1.656***	-3.626***	1.973***	-1.623***	-0.572**	**3**	-3.940	-8.887	5.498	-12.288	-2.230
2	-0.449	-0.259	0.286	-2.559***	-0.195	**2**	-0.565	-1.464	1.002	-3.142	-0.892

Small	0.114	3.949***	0.535	0.403	0.369	**Small**	0.441	6.329	1.537	0.459	0.933
CMA	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.377	-0.386***	0.552***	-0.497*	0.093	**Big**	-1.108	-2.834	5.300	-1.765	1.053
4	-0.735***	1.588***	-0.622***	1.077	0.263	**4**	-4.849	5.358	-2.639	1.311	1.002
3	1.328***	1.840***	-0.829	-0.929***	-1.799***	**3**	3.594	2.655	-1.485	-3.390	-12.846
2	-0.799***	1.202***	0.247	0.123	1.092***	**2**	-3.273	5.440	1.310	0.286	7.875
Small	0.154	-1.079***	1.774***	1.123	-1.988***	**Small**	0.561	-3.259	13.084	0.982	-12.784
OMU	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.760**	-0.644***	0.731***	0.030	0.489***	**Big**	2.023	-3.533	4.235	0.112	7.524
4	0.003	-0.656***	-0.470***	-0.256	1.799***	**4**	0.020	-3.540	-3.850	-0.989	22.522
3	-0.552	0.246***	0.109	1.368***	-0.747**	**3**	-1.396	3.167	0.744	13.119	-2.505
2	-0.260	0.440***	1.223***	-0.375	-1.417***	**2**	-0.856	4.403	9.239	-1.342	-2.975
Small	0.994***	0.890***	-1.795***	0.772	-1.064***	**Small**	5.044	2.908	-14.010	0.671	-6.422
Constant	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.026***	-0.005	-0.010**	0.002	-0.002	**Big**	-3.637	-1.089	-2.446	0.360	-0.396
4	-0.019***	0.010	-0.009	-0.006	-0.012**	**4**	-3.213	1.459	-1.566	-0.279	-2.070
3	0.007	-0.051***	-0.003	0.005	-0.019**	**3**	0.863	-3.530	-0.307	0.429	-2.411
2	-0.002	-0.013*	-0.021***	0.012	-0.025**	**2**	-0.141	-1.821	-3.129	0.614	-2.568
Small	-0.048***	-0.028***	-0.010	-0.061***	-0.008	**Small**	-4.789	-2.884	-1.557	-2.908	-1.257
R-2	**H_B|M**	**4**	**3**	**2**	**L_B|M**						
Big	0.769	0.757	0.840	0.171	0.317						
4	0.896	0.370	0.372	0.808	0.790						
3	0.339	0.915	0.874	0.695	0.666						
2	0.563	0.138	0.471	0.988	0.483						
Small	0.535	0.664	0.974	0.618	0.967						

Notes: *** and * denote statistical significance levels of 1% and 10%, respectively. Table 7 displays the estimated coefficients, t-statistics (right side), and R-square for each portfolio obtained from the multivariate WLS regression with 25 value-weighted portfolios using T5FM.

Table 7 presents the Tobin-q risk-factor augmented 5FM (T5FM) using value-weighted 25 portfolios constructed on the basis of Size-B|M ratio for PSX. The Tobin-q plus MSVPI impact on portfolio returns has been analyzed using WLS regression procedure. The findings reveal statistically significant coefficients of 21, 15, 18, 14, 16 and 17 out of 25 portfolios for MSVPI and Tobin-q risk-factor respectively based on t-stats criteria. In comparison with the liquidity augmented 5FM findings (L5FM), the profitability slightly decreases significance for portfolio returns as from 20 it declines into 14 out of 25 significant results. Remarkably, the Tobin-q factor exhibits significant impact on portfolio returns for PSX which is consistent with Azam (2022). Furthermore, the R-square ranges from 13% to 98%. The results show that WLS regression produces more valid and robust results than Azam (2021; 2022), who uses OLS regression on a nearly identical PSX dataset. The results show that WLS regression produces more valid and robust results than Azam (2021; 2022), who uses OLS regression on a nearly identical dataset from PSX. The R-square ranges from 17% to 98%.

Table 8: Momentum augmented Fama-French (2015) five-factor model (M5FM) Estimation of Coefficients based on weighted least squares (WLS) regression

RmRf	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.099***	-0.141***	0.031	0.113***	0.028*	Big	3.314	-4.912	1.402	6.823	1.687
4	-0.088***	0.030***	0.031**	0.244***	-0.151**	4	-5.794	2.846	2.106	8.530	-2.135
3	0.476***	0.061	0.102***	0.200***	0.014	3	11.128	1.110	13.366	6.644	0.317
2	0.058**	-0.000	0.692***	-0.033*	0.005	2	2.256	-0.011	2.860	-1.763	0.198
Small	0.008	-0.317***	-0.033	-0.044	0.144***	Small	0.513	-11.521	-1.361	-1.413	5.088
SMB	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-1.990***	0.397	-0.807***	-1.163**	-1.618***	Big	-3.472	0.697	-3.456	-2.486	-2.649
4	-1.063***	0.224	-0.481	-0.557	0.689	4	-4.755	0.726	-1.262	-0.855	0.809

3	0.702***	-0.409**	-0.124	1.120***	-0.236	3	5.555	-2.530	-1.399	2.723	-1.040
2	0.783	0.006	3.121**	-0.008	0.694***	2	1.111	0.009	2.027	-0.018	2.666
Small	-0.656**	1.466*	0.845*	-2.095**	0.369	Small	-2.517	1.956	1.767	-2.524	0.455
HML	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	2.851***	1.174**	0.818***	-0.231	-0.989***	Big	6.612	2.273	5.902	-0.541	-3.351
4	1.065***	2.507***	-1.442***	-0.193	1.256	4	2.650	4.847	-9.662	-0.147	1.550
3	4.791***	-1.508***	0.214*	1.494**	1.707**	3	9.162	-2.674	1.730	2.474	2.309
2	0.040	2.262***	1.503**	-1.592**	1.238***	2	0.039	2.968	2.075	-2.160	2.951
Small	-0.140	3.807***	0.252	1.302**	-1.337	Small	-0.599	3.409	1.442	2.249	-1.279
RMW	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.242	-1.141**	-0.299*	0.557***	-1.943***	Big	-0.633	-2.551	-1.652	3.761	-3.911
4	-0.025	0.559	-1.122***	-0.524	1.592	4	-0.075	0.899	-3.414	-0.536	1.262
3	2.882***	-3.091***	-1.209***	-0.656***	2.871***	3	3.977	-3.898	-2.875	-3.090	5.069
2	-1.817**	1.286***	0.825	-1.231	0.577**	2	-2.341	5.221	0.997	-1.286	2.042
Small	-1.007***	-0.254	-1.423	0.137	1.366	Small	-6.732	-0.308	-1.204	0.248	0.840
CMA	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.300	-2.847***	0.290**	0.685	1.042***	Big	-1.520	-11.300	2.008	1.367	3.398
4	0.689**	1.575***	0.665*	-0.460	1.123	4	2.095	3.212	1.680	-0.425	1.317
3	-0.246	0.185	1.295**	0.830	0.976***	3	-0.330	0.154	2.176	1.091	12.627
2	1.315***	-0.190	1.489***	-0.116	0.971***	2	5.180	-0.183	2.741	-0.181	6.056
Small	0.806**	1.934***	-1.615***	-0.400	2.695***	Small	2.204	3.359	-8.345	-0.461	3.014
WML	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-1.411***	-0.038	-0.018	-0.886**	0.347	Big	-3.668	-0.091	-0.095	-2.131	1.147
4	-0.431	-0.939***	0.495**	-1.038**	0.341	4	-0.934	-2.724	2.090	-2.370	-0.841
3	-0.727	0.101	0.088	-1.551***	-0.784	3	-0.934	0.313	1.097	-3.268	-1.077
2	-0.687**	0.853	-3.254***	-1.308*	-0.348	2	-2.176	1.450	-6.570	-1.853	-0.603
Small	0.396	-1.649***	-0.211	1.614**	-1.174	Small	1.177	-3.563	-1.190	2.171	-1.100
Constant	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.021***	-0.019**	-0.012***	-0.017***	-0.021***	Big	-3.406	-2.133	-3.937	-17.283	-27.760
4	-0.008***	-0.022***	-0.023***	-0.003***	0.036***	4	-3.127	-18.469	-20.649	-2.672	12.684
3	0.009***	-0.008***	-0.014***	-0.012***	-0.014***	3	3.167	-6.106	-34.535	-8.580	-9.187
2	-0.020***	-0.022***	-0.024***	-0.025***	-0.016***	2	-18.319	-21.528	-6.776	-22.566	-9.448
Small	-0.026***	0.000	-0.016***	-0.022***	-0.028***	Small	-27.704	0.042	-7.605	-12.466	-17.444
R-2	**H_B|M**	**4**	**3**	**2**	**L_B|M**						
Big	0.432	0.740	0.357	0.989	0.328						
4	0.590	0.430	0.663	0.842	0.308						
3	0.858	0.482	0.480	0.587	0.857						
2	0.582	0.684	0.665	0.775	0.436						
Small	0.489	0.627	0.310	0.296	0.914						

Notes: *** and * denote statistical significance levels of 1% and 10%, respectively. Table 8 displays the estimated coefficients, t-statistics (right side), and R-square for each portfolio obtained from the multivariate WLS regression with 25 value-weighted portfolios using M5FM.

Table 8 demonstrates the findings of momentum augmented 5FM (M5FM) using WLS regression technique. The results reveal that market premium presents significant findings for 17 portfolios while value and profitability show 14 portfolios coefficients statistically significant for the market. The investment premium also displays slightly improved results as 15 out of 25 portfolio coefficients show significant findings. However, the value premium demonstrates significant coefficients for 18 out of 25 portfolios. Conversely, the momentum premium shows weaker findings as 11 coefficients demonstrate significant results for PSX. Conclusively, the R-square reveals substantially improve results as it ranges from 29 percent to 98 percent. The results demonstrate that, when compared to Azam (2021; 2022), who uses OLS regression with a nearly identical

dataset from PSX, WLS regression yields more reliable and robust results. The R-square ranges from 30% to 98%.

Table 9: Liquidity and Momentum augmented Fama-French (2015) five-factor model (LM5FM) Estimation of Coefficients based on weighted least squares (WLS) regression

RmRf	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.066***	-0.115***	-0.006	-0.081***	-0.009	Big	4.511	-5.675	-0.643	-10.261	-1.010
4	-0.042	0.034**	-0.023	0.068***	-0.079***	4	-1.414	2.306	-1.476	4.643	-2.977
3	0.216***	-0.072	0.097***	-0.138***	0.015*	3	11.001	-1.632	3.197	-6.337	1.758
2	-0.090***	-0.017	-0.272***	-0.038	-0.041***	2	-3.775	-1.602	-3.560	-1.467	-3.024
Small	0.000	0.021	0.048	-0.176***	0.107***	Small	0.001	1.526	1.236	-8.246	5.656
SMB	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.697***	-1.142***	-0.889***	-1.590***	-1.410***	Big	-2.854	-4.238	-7.644	-8.467	-5.342
4	-0.143	-0.750***	-0.784***	-1.459***	-0.445***	4	-1.221	-4.857	-6.080	-14.892	-3.207
3	-0.641***	-0.668**	-0.771***	-1.136***	-0.198	3	-5.627	-2.104	-7.076	-15.490	-1.034
2	1.463***	-1.012	-0.543	-0.235	0.353	2	2.920	-1.619	-1.113	-0.701	0.689
Small	1.857***	1.725***	-1.491**	-0.019	0.610	Small	3.101	2.931	-2.378	-0.038	0.964
HML	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.076	-1.578***	-0.521***	-0.171	-0.609***	Big	0.346	-5.816	-5.624	-0.624	-3.145
4	-0.989***	0.069	-0.073	-1.168***	-0.439**	4	-3.485	0.202	-0.535	-4.084	-1.994
3	-1.146***	1.614***	1.544***	1.171***	-0.697**	3	-5.526	4.360	9.166	5.079	-2.241
2	1.708***	-0.174	0.538***	0.692	0.704	2	3.358	-0.414	2.934	1.540	1.113
Small	2.374***	1.066	0.475**	-0.857**	-2.644***	Small	8.221	1.643	2.350	-2.252	-3.535
RMW	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-1.617***	-1.774***	-0.639***	-2.750***	-0.219	Big	-7.174	-6.081	-5.386	-16.843	-0.831
4	-0.006	-0.661	-0.396**	-0.409*	-1.787***	4	-0.024	-1.593	-2.420	-1.891	-6.968
3	-1.274***	-0.648	0.799*	0.942***	-0.149	3	-4.033	-1.058	1.721	10.607	-0.771
2	-2.511***	-1.756***	-1.863***	0.026	-0.166	2	-5.632	-10.157	-7.590	0.056	-0.439
Small	-0.910***	0.482	1.221	-0.512*	-1.736	Small	-4.675	0.945	1.273	-1.786	-1.569
CMA	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.916***	2.296***	0.690***	2.557***	1.004***	Big	-4.715	13.529	5.986	10.720	7.298
4	0.350	-0.649**	0.351**	0.572***	0.336*	4	1.183	-2.315	2.294	2.914	1.912
3	3.102***	0.807	-1.242**	-0.636***	-1.503***	3	9.458	0.944	-2.093	-2.663	-9.606
2	0.973***	-1.063**	0.024	-0.206	-0.471***	2	6.287	-1.982	0.140	-0.726	-2.681
Small	-1.494**	0.179	0.144	0.689*	2.191***	Small	-2.564	0.578	0.542	1.896	3.941
IML	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-1.838***	-0.897*	-0.856***	-1.119***	-0.896**	Big	-4.655	-1.923	-4.583	-3.183	-2.279
4	-0.822*	-0.497	-0.829***	-0.435	-1.689***	4	-1.774	-1.019	-3.091	-1.167	-4.751
3	-0.249	-1.316	0.328	-0.482	-0.864	3	-0.531	-1.158	0.557	-1.007	-1.327
2	-1.189**	1.590	0.469	0.730	-1.047	2	-1.992	1.513	0.805	1.071	-1.268
Small	-1.934***	-1.877***	1.705	-1.875***	1.231***	Small	-2.767	-2.766	1.328	-3.335	3.096
WML	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.606***	-1.235***	0.685***	0.398***	0.726***	Big	4.614	-7.673	13.357	3.953	5.321
4	0.032	0.529***	0.566***	0.103	0.954***	4	0.288	2.978	4.304	0.504	5.018
3	1.142***	3.021***	0.526	1.492***	0.787***	3	7.909	4.985	1.414	6.349	3.413
2	-0.006	-0.806**	-0.514	-1.046***	0.484	2	-0.020	-2.013	-1.502	-4.208	1.620
Small	0.883***	0.721***	-0.542	1.878***	-3.789***	Small	4.451	2.751	-0.559	35.442	-7.879
Constant	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.014***	-0.007***	-0.027***	-0.012***	-0.033***	Big	-9.076	-5.089	-35.801	-16.476	-57.678
4	-0.024***	-0.029***	-0.022***	-0.022***	-0.023***	4	-14.474	-19.168	-17.836	-18.853	-23.062
3	-0.037***	-0.031***	-0.025***	-0.033***	-0.020***	3	-21.098	-8.598	-12.941	-46.302	-22.069
2	-0.014***	-0.029***	-0.026***	-0.025***	-0.019***	2	-8.275	-16.988	-14.781	-10.821	-14.610
Small	-0.027***	-0.033***	-0.013***	-0.039***	-0.025***	Small	-17.545	-17.905	-9.564	-51.119	-19.827

R-squared	H_B|M	4	3	2	L_B|M
Big	0.723	0.775	0.957	0.787	0.962
4	0.354	0.357	0.142	0.77	0.272
3	0.909	0.892	0.647	0.572	0.643
2	0.458	0.736	0.930	0.149	0.579
Small	0.841	0.523	0.540	0.986	0.830

Notes: *** and * denote statistical significance levels of 1% and 10%, respectively. Table 9 displays the estimated coefficients, t-statistics (right side), and R-square for each portfolio obtained from the multivariate WLS regression with 25 value-weighted portfolios using LM5FM.

Table 9 demonstrates the liquidity and momentum augmented 5FM (LM5FM) using WLS regression technique. The market risk-factor shows the weakest findings as compare to all models indicating 15 out of 25 portfolio exhibit significant effects on portfolio returns. Conversely, the size and value factors show 17 out of 25 portfolios statistically significant. However, the profitability premium shows 15 portfolios significant coefficients while investment and momentum both demonstrates 18 portfolios significant coefficients for the market. Conversely, the liquidity premium exhibits the weakest results as 13 out of 25 portfolios shows significant coefficients in the model for PSX. Moreover, the R-square ranges from 14 percent to 96 percent which also demonstrates weaker lowest range after T5FM findings. The results demonstrate that WLS regression generates more reliable results than Azam (2021; 2022) OLS regression with a nearly identical dataset from PSX.

Table 10: Liquidity and Tobin-q augmented Fama-French (2015) five-factor model (LT5FM) Estimation of Coefficients based on weighted least squares (WLS) regression

RmRf	H_B|M	4	3	2	L_B|M	t-stat	H_B|M	4	3	2	L_B|M
Big	-0.053**	-0.106***	0.120***	-0.022**	-0.020**	Big	-2.019	-5.380	16.326	-2.036	-1.982
4	-0.090***	0.054***	0.142***	0.146***	-0.063	4	-3.582	5.033	4.410	4.173	-1.253
3	0.014	-0.165***	-0.049***	0.005	0.066***	3	0.679	-12.069	-3.026	0.443	3.986
2	0.015	-0.010	-0.235***	-0.011	-0.015	2	0.765	-1.139	-2.851	-0.556	-1.348
Small	0.089***	-0.007	0.197***	-0.024	0.040**	Small	4.030	-0.444	5.158	-1.233	2.080
SMB	H_B|M	4	3	2	L_B|M	t-stat	H_B|M	4	3	2	L_B|M
Big	-0.775**	-1.126***	-0.795***	-0.204	-1.137***	Big	-2.117	-6.600	-6.166	-1.465	-4.552
4	-0.891***	-0.278**	0.482*	-1.249***	-1.994***	4	-8.376	-2.585	1.767	-8.604	-9.818
3	-1.291***	-0.138	-0.282***	0.058	-0.502	3	-10.838	-0.593	-4.027	1.501	-1.311
2	1.212***	0.044	-0.660	-0.202	0.148	2	2.891	0.130	-1.153	-0.547	0.355
Small	-1.050*	0.710	-1.133*	1.183**	-1.416*	Small	-1.695	1.518	-1.891	2.323	-1.758
HML	H_B|M	4	3	2	L_B|M	t-stat	H_B|M	4	3	2	L_B|M
Big	0.136	-0.680***	-0.264**	-0.430**	-0.023	Big	0.434	-3.146	-2.131	-2.328	-0.094
4	-1.101***	0.516**	-0.353	-0.269	-2.490***	4	-3.928	2.248	-1.098	-0.372	-5.057
3	1.048***	-1.025***	0.095	-0.084	1.201**	3	4.185	-5.731	0.712	-0.563	2.119
2	1.070***	0.191	-0.226	0.891**	0.116	2	2.934	0.991	-0.932	2.254	0.248
Small	0.088	0.912**	-0.435	-1.151***	-0.591	Small	0.220	1.971	-1.064	-3.061	-0.707
RMW	H_B|M	4	3	2	L_B|M	t-stat	H_B|M	4	3	2	L_B|M
Big	-0.253	-1.915***	0.034	-0.790***	-0.967***	Big	-0.802	-8.531	0.280	-6.978	-2.944
4	-0.414*	-0.066	0.937***	-0.019	-2.589***	4	-1.913	-0.234	3.081	-0.049	-5.890
3	1.999***	0.250	-0.506**	0.807***	-1.595***	3	6.435	0.926	-2.162	9.063	-4.988
2	-0.457	-0.992***	0.363	-0.448	-0.409	2	-1.503	-5.996	1.018	-1.270	-1.212
Small	-3.467***	1.004***	2.551***	-1.718***	-2.563*	Small	-14.310	2.602	3.613	-6.283	-1.703
CMA	H_B|M	4	3	2	L_B|M	t-stat	H_B|M	4	3	2	L_B|M
Big	1.176***	2.044***	1.416***	0.688***	1.890***	Big	5.796	14.249	13.521	3.641	7.600
4	-1.020***	-0.528*	1.130**	1.736***	1.993***	4	-4.462	-1.949	2.491	3.577	4.865
3	-0.100	1.654***	0.471	-0.428***	-0.076	3	-0.277	3.441	1.234	-3.853	-0.353
2	2.274***	0.241	-1.197***	-1.002***	1.911***	2	14.336	0.924	-4.637	-4.890	10.297

Small	2.522***	0.102	1.654***	-0.257	0.527	Small	5.784	0.417	6.272	-0.707	1.201
IML	H_B|M	4	3	2	L_B|M	t-stat	H_B|M	4	3	2	L_B|M
Big	-1.160*	-0.188	-0.841***	-1.577***	-1.533***	Big	-1.664	-0.520	-3.483	-7.172	-2.998
4	-1.909***	-0.550	-0.184	0.190	0.091	4	-4.379	-1.619	-0.243	0.207	0.128
3	-0.399	-0.793	-0.948***	-0.535*	-0.966	3	-0.731	-1.232	-3.026	-1.767	-0.939
2	-2.032***	-0.547	0.256	0.277	-1.207	2	-4.329	-0.964	0.409	0.377	-1.608
Small	-0.269	-2.052***	0.452	-1.674***	0.468	Small	-0.306	-3.970	0.503	-2.917	0.621
OMU	H_B|M	4	3	2	L_B|M	t-stat	H_B|M	4	3	2	L_B|M
Big	1.588***	0.348**	0.301***	1.841***	2.027***	Big	6.978	2.071	3.754	26.962	4.897
4	1.894***	0.638***	-1.743***	-1.197*	1.429***	4	11.138	2.862	-3.372	-1.664	2.820
3	0.720***	0.812	0.356	1.337***	1.029**	3	2.888	1.602	1.281	5.542	2.009
2	0.219	0.569	1.344***	-0.187	0.276	2	0.767	1.184	2.719	-0.446	1.419
Small	-3.182***	3.255***	-0.751	0.906***	-0.923***	Small	-6.177	13.630	-1.124	4.014	-4.610
Constant	H_B|M	4	3	2	L_B|M	t-stat	H_B|M	4	3	2	L_B|M
Big	0.009***	-0.026***	-0.014***	-0.017***	-0.026***	Big	4.917	-19.183	-18.828	-22.043	-24.813
4	-0.021***	-0.028***	-0.008***	-0.017***	-0.023***	4	-17.097	-21.787	-6.689	-12.027	-10.471
3	-0.042***	-0.027***	-0.015***	-0.023***	-0.009***	3	-63.083	-28.589	-18.483	-48.916	-5.723
2	-0.022***	-0.022***	-0.005**	-0.016***	-0.024***	2	-23.684	-40.291	-2.162	-31.683	-22.912
Small	-0.029***	-0.024***	-0.033***	-0.017***	-0.023***	Small	-21.591	-24.303	-18.247	-10.645	-17.593

R-squared	H_B|M	4	3	2	L_B|M
Big	0.302	0.983	0.916	0.944	0.451
4	0.623	0.390	0.372	0.722	0.528
3	0.969	0.716	0.430	0.967	0.533
2	0.692	0.251	0.209	0.915	0.531
Small	0.968	0.971	0.652	0.312	0.876

Notes: *** and * denote statistical significance levels of 1% and 10%, respectively. Table 10 displays the estimated coefficients, t-statistics (right side), and R-square for each portfolio obtained from the multivariate WLS regression with 25 value-weighted portfolios using LT5FM.

Table 10 shows the WLS regression results using liquidity and Tobin-q augmented 5FM (LT5FM). In this model, the liquidity and Tobin-q plus MSVPI impact on portfolio returns has been analyzed based on t-stats criteria. The results reveal comparatively weaker in comparison with 5FM as market, size and profitability factor demonstrate 16 out of 25 significant coefficients in the market. The value and liquidity factor display 13 and 10 out of 25 portfolio coefficients statistically significant respectively. Moreover, Tobin-q exhibits statistically significant results as 18 out of 25 portfolios. The R-square ranges from 21% to 98% which displays that both liquidity and Tobin-q contribute to the model in augmentation with 5FM for PSX. The results demonstrate that, when compared to Azam (2021; 2022), who uses OLS regression with a nearly identical dataset from PSX, WLS regression yields more reliable and robust results.

Table 11: Momentum and Tobin-q augmented Fama-French (2015) five-factor model (MT5FM) Estimation of Coefficients based on weighted least squares (WLS) regression

RmRf	H_B|M	4	3	2	L_B|M	t-stat	H_B|M	4	3	2	L_B|M
Big	0.135***	0.165***	0.310***	0.338***	-0.342***	Big	4.303	8.804	10.954	10.297	-10.007
4	0.210***	0.129***	0.034***	-0.175***	-0.011	4	4.775	6.621	2.657	-6.003	-0.184
3	0.177***	-0.168	0.046	-0.711***	-0.074*	3	5.961	-1.531	1.483	-11.053	-1.921
2	0.337***	-0.106***	-0.262	0.262***	0.056	2	9.661	-3.233	-1.301	5.081	1.324
Small	-0.034	-0.195***	0.316***	-0.024	0.025	Small	-1.110	-6.987	16.322	-1.150	1.284
SMB	H_B|M	4	3	2	L_B|M	t-stat	H_B|M	4	3	2	L_B|M
Big	-0.551	-1.359***	0.993*	-0.664	-0.593	Big	-0.687	-3.086	1.937	-1.249	-0.721
4	-0.849	-0.349	-0.610*	-0.117	0.781	4	-1.394	-1.143	-1.650	-0.181	1.160
3	0.178*	-1.343***	-1.460***	-0.862	-0.171	3	1.913	-3.923	-13.571	-0.860	-0.983
2	-0.246	1.122**	-1.502	-0.674	2.161***	2	-0.246	2.469	-1.174	-0.878	5.302

Small	-0.941	-0.836	0.139	-0.305	0.023	Small	-1.223	-1.470	0.314	-0.588	0.053
HML	H_B|M	4	3	2	L_B|M	t-stat	H_B|M	4	3	2	L_B|M
Big	-1.543***	0.755**	-0.623**	-1.043**	0.495	Big	-2.658	1.969	-2.268	-2.314	1.289
4	-0.171	0.581	-1.064***	-2.020	1.681***	4	-0.173	1.136	-7.049	-1.465	2.610
3	0.964***	3.368***	-1.847***	-3.562**	-1.865***	3	2.662	3.333	-10.686	-2.170	-3.159
2	-0.580	0.742	-1.879***	-1.423	-0.614	2	-0.386	1.631	-3.091	-1.144	-0.924
Small	1.950***	0.832	-0.329*	-0.029	-0.909	Small	4.570	0.980	-1.814	-0.075	-1.618
RMW	H_B|M	4	3	2	L_B|M	t-stat	H_B|M	4	3	2	L_B|M
Big	-2.339***	-2.116***	-1.497***	-2.629***	0.887	Big	-3.692	-5.064	-3.722	-10.933	1.421
4	-0.226	-1.109*	-1.564***	1.005	-0.948	4	-0.277	-1.730	-4.978	1.044	-0.953
3	-3.044***	-0.042	-0.994	3.319***	-1.595***	3	-6.111	-0.029	-1.604	8.048	-3.239
2	-1.458	-0.192	-2.383***	-2.585*	2.851***	2	-1.217	-1.388	-3.362	-1.703	5.287
Small	0.405*	0.267	2.067*	0.629	-0.699	Small	1.934	0.395	1.667	1.649	-0.806
CMA	H_B|M	4	3	2	L_B|M	t-stat	H_B|M	4	3	2	L_B|M
Big	-2.468***	0.689***	-0.357	1.039*	-0.121	Big	-10.392	3.041	-0.932	1.762	-0.299
4	-1.480**	3.705***	0.397	0.484	-0.987	4	-1.974	7.671	1.050	0.418	-1.417
3	2.867***	-3.361	1.072	8.022***	0.403**	3	6.188	-1.474	1.174	3.926	2.183
2	2.866***	2.057***	0.257	0.144	1.873***	2	5.564	3.205	0.432	0.124	6.680
Small	-0.588	-0.544	0.606***	-0.718	1.144**	Small	-0.587	-1.059	2.783	-1.299	2.412
WML	H_B|M	4	3	2	L_B|M	t-stat	H_B|M	4	3	2	L_B|M
Big	-2.810***	0.748*	-0.763*	-0.279	1.206*	Big	-5.005	1.898	-1.766	-1.112	1.959
4	0.499	-0.189	0.558**	0.101	-1.628***	4	0.469	-0.652	2.229	0.159	-4.097
3	1.776***	2.914***	-0.168	-2.975	0.507	3	4.037	3.279	-0.748	-1.517	1.080
2	-2.470***	-1.024*	2.018***	0.315	-2.589***	2	-7.315	-1.653	3.966	0.227	-3.307
Small	-1.020	-0.462	-1.313***	-0.285	-1.499**	Small	-0.843	-1.438	-4.048	-0.739	-2.530
OMU	H_B|M	4	3	2	L_B|M	t-stat	H_B|M	4	3	2	L_B|M
Big	0.182	-0.761***	-0.396*	0.765	1.092***	Big	0.504	-5.946	-1.795	1.521	2.728
4	-0.291	-0.452**	-0.197*	-2.609***	2.042***	4	-0.791	-2.517	-1.668	-7.256	7.710
3	-2.647***	-1.229**	-0.651***	-1.044	-2.038***	3	-10.538	-1.988	-2.773	-0.816	-7.162
2	0.669*	0.293	-1.289***	-0.305	0.659	2	1.904	0.575	-4.192	-0.692	1.446
Small	1.480***	1.710***	0.368	0.480**	-0.028	Small	2.947	8.091	0.815	2.337	-0.203
Constant	H_B|M	4	3	2	L_B|M	t-stat	H_B|M	4	3	2	L_B|M
Big	0.006**	-0.018***	-0.007***	0.004*	-0.016***	Big	2.404	-15.469	-3.627	1.701	-19.937
4	0.005	-0.004*	-0.023***	-0.028***	0.019***	4	0.820	-1.926	-22.097	-13.055	9.446
3	-0.020***	-0.021***	-0.000	-0.002	-0.013***	3	-10.051	-6.007	-0.150	-0.455	-6.792
2	0.024***	-0.029***	-0.011***	-0.058***	0.009***	2	9.359	-14.399	-2.618	-23.724	3.030
Small	-0.006***	-0.025***	-0.032***	-0.017***	-0.021***	Small	-9.923	-12.685	-26.673	-23.840	-12.482
R-squared	H_B|M	4	3	2	L_B|M						
Big	0.836	0.721	0.799	0.973	0.864						
4	0.390	0.728	0.832	0.667	0.489						
3	0.798	0.901	0.896	0.700	0.946						
2	0.647	0.272	0.767	0.492	0.861						
Small	0.932	0.954	0.800	0.676	0.482						

Notes: *** and * denote statistical significance levels of 1% and 10%, respectively. Table 11 displays the estimated coefficients, t-statistics (right side), and R-square for each portfolio obtained from the multivariate WLS regression with 25 value-weighted portfolios using MT5FM.

Table 11 represents the momentum and Tobin-q augmented 5FM (MT5FM) using WLS regression technique. Using t-statistics criteria, the market risk-factor reveals significant coefficients for 17 out of 25 portfolios showing the existence of CAPM in the market. The size factor demonstrates the weakest and insignificant results as only 8 out of 25 portfolios coefficients show significant findings. Conversely, the value, profitability, and momentum show similar results as 14 out of 25 portfolios coefficients demonstrate significant findings for the market. However, the investment-factor coefficients show 13 out of 25 significant results as well as Tobin-

q indicates highly statistically significant coefficients after market risk-factor, confirming by the t-statistic values which are greater than two also shows consistency with Azam (2022). The results demonstrate that, when compared to Azam (2021; 2022), who uses OLS regression with a nearly identical dataset from PSX, WLS regression yields more reliable and robust results. The R-squared ranges from 39% to 97% which indicating that Tobin-q augmented 5FM explains better results in the market.

Table 12: Liquidity, Momentum and Tobin-q augmented Fama-French (2015) five-factor model (LMT5FM) estimation of coefficients based on weighted least squares (WLS) regression

RmRf	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.066**	-0.027**	0.076***	-0.049***	0.070***	Big	2.458	-2.233	5.478	-8.606	4.804
4	0.024	0.011	0.036**	0.106***	-0.060*	4	1.417	0.734	1.997	4.291	-1.872
3	-0.031	-0.126***	-0.232***	0.027**	0.049***	3	-1.319	-3.077	-9.875	2.122	4.209
2	-0.113***	-0.000	-0.039	0.087***	-0.035***	2	-2.948	-0.050	-0.384	3.904	-2.906
Small	0.102***	-0.015	-0.036	-0.051***	-0.100***	Small	5.716	-1.330	-0.864	-2.806	-3.754
SMB	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.659*	-0.516***	-1.171***	-0.312**	-0.882***	Big	-1.921	-4.050	-7.739	-2.336	-3.575
4	-0.545***	-0.443***	0.576***	0.080	-1.227***	4	-6.241	-3.601	3.927	0.840	-9.790
3	-0.835***	0.110	-0.573***	-0.067	0.395***	3	-7.286	0.299	-4.211	-1.501	2.746
2	1.748**	0.952**	-0.069	0.989***	0.348	2	2.202	2.526	-0.097	2.698	0.880
Small	1.367*	0.575	-0.566	1.063*	0.796	Small	1.730	1.234	-0.899	1.815	1.461
HML	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.208	-0.016	-0.466***	-0.784***	-0.336	Big	0.724	-0.092	-3.773	-4.139	-1.279
4	0.755***	0.228	0.429**	1.091**	-1.162***	4	4.362	0.727	2.298	2.263	-4.165
3	-0.569**	2.444***	0.572***	-0.036	-0.083	3	-2.533	7.323	3.123	-0.226	-0.351
2	1.632**	0.217	0.596***	-0.909**	0.809*	2	2.139	1.216	2.867	-2.238	1.693
Small	0.227	1.167**	2.170***	-0.540	-1.749***	Small	0.809	2.278	6.387	-1.346	-2.717
RMW	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.761**	-0.783***	-0.782***	-1.658***	1.094***	Big	-2.557	-4.220	-6.014	-12.621	4.020
4	-1.108***	-0.848**	1.003***	-0.231	-1.163***	4	-7.260	-2.247	4.508	-0.958	-3.961
3	-0.586**	-0.508	0.112	0.563***	0.708***	3	-2.344	-1.098	0.357	5.862	5.960
2	-0.658	-0.435***	2.245***	-0.919**	0.581**	2	-1.069	-3.165	5.082	-2.535	2.051
Small	1.213***	-0.384	-0.984	-2.297***	-1.910**	Small	4.542	-0.948	-1.039	-8.259	-2.166
CMA	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.811***	1.902***	1.451***	1.159***	1.224***	Big	4.446	14.159	10.009	6.295	3.460
4	-0.283**	1.226***	-0.520**	-0.878***	0.629***	4	-2.159	3.853	-2.583	-3.075	3.138
3	0.680**	2.620***	1.071**	-0.479***	-0.071	3	2.057	3.483	2.317	-4.573	-0.430
2	0.993***	0.588*	0.269	-0.195	-0.517***	2	6.061	1.953	0.977	-0.814	-3.230
Small	2.554***	0.615**	-2.583***	-0.017	-0.642	Small	3.946	2.293	-9.717	-0.040	-1.394
IML	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-0.936	-0.602**	-0.629**	-1.286***	-1.528***	Big	-1.440	-2.270	-2.303	-5.710	-3.083
4	-0.959***	-1.475***	-1.421***	-0.976	0.229	4	-3.481	-2.890	-3.263	-1.460	0.528
3	-1.159**	-1.257	-1.339***	-0.468	-0.738*	3	-2.585	-1.173	-2.629	-1.579	-1.691
2	-0.670	-1.670***	0.965	-0.630	-0.457	2	-0.669	-2.731	1.108	-0.901	-0.711
Small	-0.741	-1.202**	-0.942	-1.560**	-0.759**	Small	-0.777	-2.205	-0.761	-2.419	-2.097
WML	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	-1.572***	-0.001	0.744***	-0.219**	0.301	Big	-11.716	-0.006	6.727	-1.997	1.444
4	0.131	1.048***	-0.945***	-0.957***	-1.015***	4	1.220	8.892	-5.766	-4.988	-6.015
3	0.692***	2.590***	1.341***	0.092	-0.198	3	4.240	8.918	7.529	1.014	-1.646
2	-1.635***	-0.490***	-1.030***	0.474**	0.591**	2	-5.212	-4.499	-3.661	2.467	2.446
Small	0.010	-0.622***	-1.235	0.534***	-2.033***	Small	0.020	-3.086	-1.511	3.900	-4.275
OMU	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	2.034***	-0.181	-0.247**	1.071***	0.832*	Big	10.052	-1.057	-1.982	8.176	1.895
4	-0.232*	0.139	1.129***	2.409***	0.553**	4	-1.890	0.578	4.208	5.397	2.028

3	1.401***	1.611**	-0.271	1.101***	0.791***	3	5.334	2.262	-0.739	4.776	5.136
2	0.319	1.976***	-0.778*	0.457	-0.207*	2	0.797	3.450	-1.664	1.000	-1.946
Small	-0.524	0.815***	3.158***	0.858***	2.144***	Small	-0.640	2.606	7.910	3.813	7.642
Constant	**H_B|M**	**4**	**3**	**2**	**L_B|M**	**t-stat**	**H_B|M**	**4**	**3**	**2**	**L_B|M**
Big	0.009***	-0.013***	-0.021***	-0.019***	-0.025***	Big	6.531	-6.120	-23.065	-29.337	-15.698
4	-0.007***	-0.036***	-0.010***	-0.029***	-0.021***	4	-5.627	-22.898	-5.883	-17.411	-15.098
3	-0.045***	-0.036***	-0.024***	-0.022***	-0.021***	3	-24.050	-15.008	-21.470	-32.330	-17.934
2	-0.023***	-0.020***	-0.012***	-0.032***	-0.025***	2	-32.890	-26.685	-6.106	-19.835	-22.214
Small	-0.026***	-0.028***	-0.017***	-0.029***	-0.013***	Small	-23.396	-31.570	-10.349	-14.522	-7.683
R-2	**H_B|M**	**4**	**3**	**2**	**L_B|M**						
Big	0.710	0.519	0.920	0.783	0.807						
4	0.819	0.709	0.470	0.665	0.533						
3	0.711	0.972	0.389	0.983	0.890						
2	0.823	0.753	0.919	0.709	0.550						
Small	0.751	0.582	0.973	0.688	0.305						

Notes: *** and * denote statistical significance levels of 1% and 10%, respectively. Table 12 displays the estimated coefficients, t-statistics (right side), and R-square for each portfolio obtained from the multivariate WLS regression with 25 value-weighted portfolios using LMT5FM.

Table 12 demonstrates the LMT5FM using 25 portfolio returns on PSX. The market-risk factor shows 18 out of 25 portfolios statistically significant coefficients. Similarly, size, value, profitability and investment factors demonstrate 17, 16, 19 and 20 out of 25 portfolios statistically significant results respectively. However, liquidity factor shows moderately significant findings as 14 out of 25 portfolios coefficients are significant. Although, momentum and Tobin-q coefficients exhibit 18 and 19 out of 25 portfolios significant in explaining the portfolio returns for PSX. Furthermore, considering the R-square range, the LMT5FM is the second best model as L5FM shows the substantial improved in the market. The results demonstrate that, when compared to Azam (2021; 2022), who uses OLS regression with a nearly identical dataset from PSX, WLS regression yields more reliable and robust results.

Robustness Tests

1. Model Performance Test and Multicollinearity Test

Table 13: Gibbons, Ross & Shanken test (Wald Version) and Multicollinearity Test

Model	Wald Version	P-value	F-Value	Prob>F	Variable	VIF	1/VIF
5FM	92.721796	1.024e-09	28.06	0.0000	WML	2.76	0.362095
L5FM	100.54798	5.072e-11	62.72	0.0000	OMU	2.15	0.464777
T5FM	84.139269	2.527e-08	38.09	0.0000	IML	2.04	0.489967
M5FM	70.185812	3.611e-06	30.32	0.0000	HML	1.34	0.746236
LM5FM	124.61111	3.378e-15	66.52	0.0000	CMA	1.28	0.780131
LT5FM	105.83857	6.404e-12	51.13	0.0000	SMB	1.2	0.830618
MT5FM	66.620528	.00001207	31.84	0.0000	RMW	1.13	0.882317
LMT5FM	118.94854	3.392e-14	53.35	0.0000	RmRf	1.02	0.98427
					Mean VIF	**1.62**	

Table 13 displays the findings of GRS (Wald Version) test and VIF test to further test the robustness checks. The left hand side shows GRS test results while the right three columns show the multicollinearity test results.

The GRS (Wald version) test findings and multicollinearity test findings are presented in table 13. The findings reveal that based on F-statistics all the models qualify the test but the most suitable model is LM5FM while the

second suitable model for the market is LMT5FM. The right hand side shows the vif test for multicollinearity which indicates that no one value is greater than 5. Thus, the findings conclude that liquidity and momentum augmented 5FM explains the portfolio returns more efficiently for the PSX.

2. Robustness of our Findings

The final decision can be drawn based on the below table which displays the overall models and particularly every factor (premium) used in the study along with R-square ranges as follows:

Table 14: Conclusive findings of the study

Model	RmRf	SMB	HML	RMW	CMA	IML	WML	OMU	R-Sq. (Range)
5FM	20	16	15	20	19	-	-	-	15-94
L5FM	19	15	19	20	21	17	-	-	44-98
T5FM	21	15	18	14	16	-	-	17	13-98
M5FM	17	14	18	14	15	-	11	-	29-98
LM5FM	15	17	17	15	18	13	18	-	14-96
LT5FM	16	16	13	16	18	10	-	18	21-98
MT5FM	17	8	14	14	13	-	14	16	27-95
LMT5FM	18	17	16	19	20	14	18	19	30-98
Average	**17.88**	**14.75**	**16.25**	**16.5**	**17.5**	**13.5**	**15.25**	**17.5**	

Notes: Table 14 presents the significant coefficients out of 25 portfolios for all factors used in the study. The right side shows R-squared percentage range from lowest to highest for all models. The last row shows the average significant factor coefficients for all factors used in the study.

Table 14 shows the results of eight different models for predicting stock market returns to check the robustness of the study core findings. The models are labeled 5FM, L5FM, T5FM, M5FM, LM5FM, LT5FM, MT5FM and LMT5FM the abbreviations in the columns refer to the different factors used in each model. RmRf indicates the excess market returns, SMB is the small minus big factor, HML is the high minus low factor, RMW is the robust minus weak factor, CMA is the conservative minus aggressive factor, IML is the illiquidity minus liquidity factor, WML is the winner minus loser factor, and OMU is the overvalued minus undervalued factor. The R-Sq. (Range) column shows the range of the model's R-squared value, which indicates how well the model fits the data. The average row shows the average of the values in the preceding rows.

Conclusions

This study extends the empirical literature on augmenting Tobin-q, multidimensional liquidity and momentum Fama-French five-factor model using Pakistan Stock Exchange. Using the time-spans of 354 months, between July 1993 and December 2022, this study examines the market and the performance of asset pricing models using 522 financial and non-financial firms' data. Using five to eight-factor asset pricing models, the weighted least squares (WLS) regression technique is utilised to study the link between liquidity, momentum, Tobin-q risk factors, as well as investment, profitability, value and size anomalies and portfolio stock returns.

The results reveal that on average market premium demonstrates statistically significant coefficients for the market using WLS regression procedure. Though the size premium reveals significant findings as well. Moreover, the value and profitability premiums show significant coefficients almost similarly. The investment and Tobin-q show similar significant coefficients on average using multiple asset pricing models for the market.

Conversely, the liquidity premium shows weaker findings as compare to all other factors for the market which show inconsistent with Azam and Naveed (2021).

The results also indicate that the size premium is significant for the two factors of size and Tobin-q. The value premium reveals significant coefficients for the two factors of size and profitability. Moreover, the profitability premium shows significant coefficients for the two factors of size and investment. The investment premium reveals significant coefficients for the two factors of size and liquidity. Lastly, the liquidity premium reveals significant coefficients for the two factors of size and value. Overall, the results demonstrate that the size, value, profitability and investment premiums are significant and demonstrate strong results for the market. The liquidity premium is the only factor that shows weaker results on average for the market. Furthermore, the findings of this research will contribute to the literature by providing evidence of the importance of the various factors on asset pricing in the Pakistan stock market. The findings of this study will also be useful to investors and policymakers in making investment decisions and developing policies. Furthermore, the findings of this study can be used to inform future research on asset pricing and the Pakistan stock market. Investors should pay close attention to the five-factor premia, particularly the Tobin-q premium, which produces significant results in the PSX.

The future potential research can be focused on constructing portfolios based on firms' sales growth as revealed by Kalim, Saeed, and Kamil (2023). Though, the conditional version of FF5FM is another alternative option for future study using PSX dataset. There are various potential risk-factors such as downside risk, leverage, price-to-earnings ratio, human-capital, etc. which can also be investigated into frontier equity market to analyze their robustness in the market.

Funding: The author did not receive support from any organization for the submitted work. The authors have no relevant financial or non-financial interests to disclose.

Conflicts of Interest: The author has no competing interests to declare that are relevant to the content of this article. "The author declares no conflict of interest." Furthermore, "The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results".

References

Ahmad, N., Shah, F. N., Ijaz, F., & Ghouri, M. N. (2023). Corporate income tax, asset turnover and Tobin's Q as firm performance in Pakistan: Moderating role of liquidity ratio. *Cogent Business & Management*, *10*(1), 2167287.

Ali, F., Wang, M., Ali, I., & Ali, S. T. (2020). Does Demutualization Spur Liquidity?. *SEISENSE Journal of Management*, *3*(1), 15-26.

Atilgan,Y., K. Demirtas,O., A. Gunaydin, D. and Kirli, I. (2022). Momentum and Downside Risk in Emerging Markets. *The Journal of Portfolio Management Emerging Markets, 48*(8), 44-58. DOI: https://doi.org/10.3905/jpm.2022.1.369

Azam, M. (2023). Fama and French (2015) five-factor model using SEM with a Mediating Role of Liquidity: Evidence from Pakistan. *Journal of Social Sciences and Management Studies*, *2*(4), 15–33. https://doi.org/10.56556/jssms.v2i4.437

Azam, M., & Azeem, N. (2021). Does COVID-19 Affect Stock Market Volatility?. *International Journal of Business and Management Sciences*, *2*(2), 57-66.

Azam, M., (2022a). The CAPM Revisited Using Structural Equation Model: Mediating Role of GDP-Growth in Pakistan Stock Exchange. *Journal of Managerial Sciences*, *16*(1), 93-108.

Azam, M. (2022b). Selectivity and Market-Timing Performance of Mutual Funds Using Human-Capital Augmented Various Asset-Pricing Models: Evidence from Pakistan. *International Journal of Management Research and Emerging Sciences*, *12*(3), 1-31.

Azam, M. (2022c). An empirical investigation of Tobin's-Q augmented various Asset Pricing Models: Evidence from Pakistan. *Journal of Social Sciences and Management Studies*, *1*(4), 1–22. https://doi.org/10.56556/jssms.v1i4.293

Azam, M., & Ilyas, J. (2011). An empirical comparison of CAPM and Fama-French Model: A case study of KSE. *Interdisciplinary Journal of Contemporary Research in Business*, *2*(12), 415-426.

Azam, M., & Naveed (2021). Multidimensional Liquidity (Liu, 2006) and Momentum (Carhart, 1997) augmented Fama & French (2015) five-factor model: Evidence from Pakistan. *International Journal of Business and Management Sciences*, *2*(3), 108-130.

Barber, B. M., & Lyon, J. D. (1997). Firm size, book-to-market ratio, and security returns: A holdout sample of financial firms. *The Journal of Finance*, *52*(2), 875-883.

Banz, R. W. (1981). The Relationship Between Return and Market Value of Common Stocks. *Journal of Financial Economics*, 9, 3-18.

Basu, S. (1977). Investment performance of common stocks in relation to their price-earnings ratios: A test of the efficient market hypothesis. *The journal of Finance*, *32*(3), 663-682.

Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of finance*, *52*(1), 57-82.

Chan, C., Hamao, Y., Lakonishok, J., & Louis, K. (1991). Fundamentals and Stock Returns in Japan. *The Journal of Finance*, 46(5), 1739-1764.

Chiah, M., Chai, D., Zhong, A., & Li, S. (2016). A Better Model? An empirical investigation of the Fama–French five-factor model in Australia. *International Review of Finance*, *16*(4), 595-638.

Demir, I., Muthuswamy, J., & Walter, T. (2004). Momentum returns in Australian equities: The influences of size, risk, liquidity and return computation. *Pacific-Basin Finance Journal*, *12*(2), 143-158.

Dirkx, P., & Peter, F. J. (2020). The Fama-French five-factor model plus momentum: Evidence for the German market. *Schmalenbach Business Review*, *72*, 661-684.

Durand, R. B., Lim, D., & Zumwalt, J. K. (2011). Fear and the Fama-French factors. *Financial Management*, *40*(2), 409-426.

Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of financial economics*, *33*(1), 3-56.

Fama, E. F., & French, K. R. (2015). A five-factor asset pricing model. *Journal of financial economics*, *116*(1), 1-22.

Fama, E. F., & MacBeth, J. D. (1973). Risk, return, and equilibrium: Empirical tests. *Journal of political economy*, *81*(3), 607-636.

Gaytán Cortés, J. (2023). Equilibrium prices of the titles: Sharpe and the Securities Valuation Model (CAPM). *Mercados y negocios*, *24*(49), 83-94.

Hua, Z. (2022). A Comparative Analysis of the Fama-French Five-Factor Model: Based on the Data of China's SME Board. *Frontiers in Business, Economics and Management*, *4*(2), 34-42.

Hurn, S., & Pavlov, V. (2003). Momentum in Australian stock returns. *Australian Journal of Management*, *28*(2), 141-155.

Jegadeesh, N. (1990). Evidence of Predictable Behaviour of Security Returns. *The Journal of Finance*, 45(3), 881-898.

Jegadeesh, N., & Titman, S. (1993). Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *The Journal of Finance*, 48(1), 65-91.

Kalim, S., Saeed, A., & Kamil, M. (2023). Does Working Capital Management Contribute to the Profitability of Manufacturing Sector? Evidence from Pakistan. *International Journal of Business and Economic Affairs*, *8*(1), 18-25. https://doi.org/10.24088/IJBEA-2023-81002

Khan, S., Wahid, F., Rahim, A., Ali, A., & Ahmad, A. (2021). Investigating the Momentum Effect in the Merging Market: Evidence from Pakistan. *Global Business Review*, 0972150921991506.

Lintner, J. (1965). Security prices, risk, and maximal gains from diversification. *The journal of finance*, *20*(4), 587-615.

Liu, W. (2006). A liquidity-augmented capital asset pricing model. *Journal of financial Economics*, *82*(3), 631-671.

Lohano, K., & Kashif, M. (2018). Testing asset pricing models on the Pakistan Stock Exchange. *IBA Business Review*, *13*(2), 1-19.

Mossin, J. (1966). Equilibrium in a capital asset market. *Econometrica: Journal of the econometric society*, 768-783.

Oyedeko, Y. O., Mamidu, A. I., & Kolawole, O. S. (2023). Pricing Higher-Order Moment Systematic Risks in the Nigerian Stock Market: Empirical Analysis from Moment-CAPM, Moment-FF3F and Moment-FF5F. *EuroEconomica*, *42*(1), 62-77.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The journal of finance*, *19*(3), 425-442.

Shi, Z. (2023). Fama-French Five-factor Model under Liquidity Adjustment--An Empirical Study Based on China's A-share Market.

Stork, P. A. (2008). Momentum effects in the largest Australian and New Zealand shares. *INFINZ Journal*, 30-33.

Tobin, J. (1969). A general equilibrium approach to monetary theory. *Journal of money, credit and banking*, *1*(1), 15-29.

Vergara-Fernández, M., Heilmann, C., & Szymanowska, M. (2023). Describing model relations: The case of the capital asset pricing model (CAPM) family in financial economics. *Studies in History and Philosophy of Science*, *97*, 91-100.